

## Lambda and Anti-Lambda Hypernuclei in Relativistic Mean-field Theory

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Several aspects about  $\Lambda$ -hypernuclei in the relativistic mean field theory, including the effective  $\Lambda$ -nucleon coupling strengths based on the successful effective nucleon-nucleon interaction PK1, hypernuclear magnetic moment and  $\bar{\Lambda}$ -hypernuclei, have been presented. The effect of tensor coupling in  $\Lambda$ -hypernuclei and the impurity effect of  $\bar{\Lambda}$  to nuclear structure have been discussed in detail.

**Keywords:** Lambda and Anti-Lambda, Hypernuclei, Relativistic mean field, Spin symmetry

### 1. Introduction

Since the first discovery of  $\Lambda$ -hypernuclei by observing cosmic-rays in emulsion chambers,<sup>1</sup> lots of efforts have been devoted to study hypernuclei. Using a variety of hypernucleus production reactions and coincidence measurement techniques, data on the single- $\Lambda$ <sup>2-12</sup> and double- $\Lambda$  hypernuclei<sup>13-17</sup> have been accumulated. With the additional degree of freedom of strangeness, hyperons can penetrate into dense nuclear matter inaccessible to proton and neutron. In astrophysics, hyperons also play a significant role in the formation and thermal structure evolution of neutron stars.<sup>18,19</sup> Prospect for production of neutron halo hypernuclei has been made via  $(K^-, \pi^+)$  reaction,<sup>20</sup> which may be helpful to form neutron halo.

On the theoretical side, non-relativistic few-body model and shell model

as well as the Skyrme-Hartree-Fock theory have been successfully used to describe single- $\Lambda$  and double- $\Lambda$  hypernuclei.<sup>21–23</sup> Moreover, the relativistic mean field (RMF) theory, which was one of the most successful approaches for ordinary nuclei,<sup>24–27</sup> has also been applied to describe the structure of nuclei with single- $\Lambda$  or multi- $\Lambda$  and other strange baryons systems.<sup>28–38</sup> Particularly, with the relativistic continuum Hartree-Bogoliubov (RCHB) theory, which has been successfully used to describe the giant halos in exotic Zr and Ca isotopes,<sup>39,40</sup> the hyperon halo in Carbon hypernuclei and neutron halo in Calcium hypernuclei have been predicted.<sup>41,42</sup>

Motivated by the accumulated data of  $\Lambda$  binding energy and spin-orbit splitting, recently the effective  $\Lambda$ -nucleon coupling strengths based on the successful effective nucleon-nucleon interaction PK1 have been proposed with microscopic correction for the center-of-mass motion. Here the new effective  $\Lambda$ -nucleon coupling strengths and calculated results for hypernuclear magnetic moment and spin symmetry in single  $\bar{\Lambda}$  spectra will be presented and the effect of tensor coupling in  $\Lambda$ -hypernuclei as well as the impurity effect of  $\bar{\Lambda}$  will be discussed in detail.

## 2. Brief introduction of the RMF theory for hypernuclei

The starting point of the RMF theory is a standard Lagrangian density  $\mathcal{L}$ , in which nucleons are described as Dirac particles that interact via the exchange of scalar  $\sigma$ , vector  $\omega$ , and isovector-vector  $\vec{\rho}$  mesons as well as photon.<sup>27</sup> For hypernuclei system, the Lagrangian density  $\mathcal{L}$  can be written into two parts,

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_Y, \quad (1)$$

where  $\mathcal{L}_0$  is the standard Lagrangian density.<sup>27</sup> The Lagrangian density  $\mathcal{L}_Y$  for hyperon  $Y$  ( $\Lambda$  or  $\bar{\Lambda}$ ) is given by,

$$\mathcal{L}_Y = \bar{\psi}_Y (i\gamma^\mu \partial_\mu - m_Y - g_{\sigma Y} \sigma - g_{\omega Y} \gamma^\mu \omega_\mu) \psi_Y + \frac{f_{\omega YY}}{4m_\Lambda} \bar{\psi}_Y \sigma^{\mu\nu} \Omega_{\mu\nu} \psi_Y \quad (2)$$

where  $m_Y$  is the mass of hyperon, and  $g_{\sigma Y}$ ,  $g_{\omega Y}$  are the coupling strengthes of hyperon and mesons. The last term in (2) is due to the tensor coupling between hyperon and  $\omega$  field, where the field tensor  $\Omega_{\mu\nu}$  for the  $\omega$ -meson is defined as  $\Omega_{\mu\nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu$ .

Restricted to mean field and no-sea approximation, following the standard procedure,<sup>24</sup> one can obtain the equations of motion for baryons (B) (nucleon (N) and hyperon (Y)) and mesons respectively, i.e., the Dirac and Klein-Gordon equations.

The Dirac equation for the hyperon is,

$$\left[ \alpha \cdot \mathbf{p} + (m_Y + S_Y) + \gamma_\mu V_Y^\mu - \frac{f_{\omega YY}}{2m_Y} \sigma_{\mu\nu} \partial^\mu \omega^\nu \right] \psi_{Yi}(\mathbf{r}) = \epsilon_i \psi_{Yi}(\mathbf{r}), \quad (3)$$

with the vector potential  $V_Y^\mu = g_{\omega Y} \omega^\mu$  and scalar potential  $S_\Lambda = g_{\sigma Y} \sigma$ . For the  $\omega$  meson, the corresponding Klein-Gordon equation reads,

$$(-\nabla^2 + m_\omega^2) \omega_\mu = \sum_B g_{\omega B} j_\mu^B - c_3 \omega_\nu \omega^\nu \omega_\mu - \frac{f_{\omega YY}}{2m_Y} j_\mu^{T,Y}, \quad (4)$$

where the baryon current  $j_\mu^B$  and tensor current  $j_\mu^{T,Y}$  have been respectively defined as,

$$j_\mu^B = \sum_i \bar{\psi}_{Bi} \gamma_\mu \psi_{Bi}, \quad (5)$$

$$j_\mu^{T,Y} = \sum_i \partial^\nu (\bar{\psi}_{Yi} \sigma_{\mu\nu} \psi_{Yi}). \quad (6)$$

More details can be found in Ref.<sup>27</sup>

### 3. New hyperon-nucleon parametrization

The RMF theory has made great success in the description of ordinary nuclei with an universal effective nucleon-nucleon interaction<sup>43–45</sup> determined by fitting the nuclear observables such as binding energy, charge radii, etc. As PK1 is one of the most successful effective nucleon-nucleon interaction, it is natural to extend PK1 for the description of hypernuclei.

For  $\Lambda$ -nucleon effective interaction, there are four additional parameters  $m_\Lambda$ ,  $g_{\sigma\Lambda}$ ,  $g_{\omega\Lambda}$  and  $f_{\omega\Lambda\Lambda}$ . The mass of  $\Lambda$  is usually fixed to the experimental value  $M_\Lambda = 1115.6$  MeV. As suggested in Ref.<sup>46</sup> the tensor  $\omega$ - $\Lambda$  is adopted as  $R_{\omega\Lambda\Lambda} = f_{\omega\Lambda\Lambda}/g_{\omega\Lambda} = 1$ . The other two parameters are usually determined by fitting the single- $\Lambda$  binding energy and /or the spin-orbit splitting.

In Ref.<sup>47</sup> a new effective hyperon-nucleon interaction Y1, based on the effective nucleon-nucleon interaction PK1, has been developed and labeled as PK1-Y1 by fitting the single- $\Lambda$  binding energies of hypernuclei  $^{12-14}_\Lambda\text{C}$ ,  $^{15}_\Lambda\text{N}$ ,  $^{16}_\Lambda\text{O}$ ,  $^{28}_\Lambda\text{Si}$ ,  $^{32}_\Lambda\text{S}$ ,  $^{40}_\Lambda\text{Ca}$ ,  $^{51}_\Lambda\text{V}$ ,  $^{89}_\Lambda\text{Y}$ ,  $^{139}_\Lambda\text{La}$  and  $^{208}_\Lambda\text{Pb}$  as well as the spin-orbit splittings in  $^9_\Lambda\text{Be}$  and  $^{13}_\Lambda\text{C}$ . The ratio of the meson- $\Lambda$  coupling strengthes to meson-nucleon ones, i.e.,  $R_\sigma = g_{\sigma\Lambda}/g_{\sigma N}$  and  $R_\omega = g_{\omega\Lambda}/g_{\omega N}$ , thus obtained are  $R_\sigma = 0.580$  and  $R_\omega = 0.620$ . The parameters of effective  $\Lambda$ -nucleon interactions PK1-Y1 is shown in Table 1 in comparison with

TM1-B,<sup>45</sup> NLSH-A, and NLSH-B.<sup>48</sup> The root-mean-squared (rms) deviation  $\Delta$  and the relative ones  $\chi$  for single- $\Lambda$  binding energies and the  $\Lambda$  spin-orbit splitting of  $p$  state in  ${}^9_\Lambda\text{Be}$  and  ${}^{13}_\Lambda\text{C}$  are also presented, where

$$\Delta \equiv \sqrt{\frac{1}{N} \sum_{i=1}^N (\mathcal{O}_i^{\text{exp.}} - \mathcal{O}_i^{\text{theo.}})^2} \text{ and } \chi \equiv \sqrt{\frac{1}{N} \sum_{i=1}^N \frac{(\mathcal{O}_i^{\text{exp.}} - \mathcal{O}_i^{\text{theo.}})^2}{(\mathcal{O}_i^{\text{exp.}})^2}}.$$

Table 1. The parameters of effective  $\Lambda$ -nucleon interactions PK1-Y1, TM1-B, NLSH-A, and NLSH-B as well as the corresponding root-mean-squared (rms) deviation  $\Delta$  and the relative ones  $\chi$  for single- $\Lambda$  binding energies of candidate hypernuclei ( $\Delta_b$ ,  $\chi_b$ ) and the  $\Lambda$  spin-orbit splitting of  $p$  state in  ${}^9_\Lambda\text{Be}$  and  ${}^{13}_\Lambda\text{C}$  ( $\Delta_p$ ,  $\chi_p$ ).

Sets	PK1-Y1	TM1-B	NLSH-A	NLSH-B
$R_\sigma, R_\omega$	0.580, 0.620	0.468, 0.485	0.621, 0.667	0.490, 0.512
$R_{\omega\Lambda\Lambda}$	1.0	1.21	1.0	0.616
$\Delta_b$	0.851	1.229	1.200	0.906
$\chi_b (10^{-2})$	5.419	6.164	7.539	5.470
$\Delta_p$	0.058	0.103	0.100	0.257
$\chi_p$	0.391	0.915	0.682	1.698

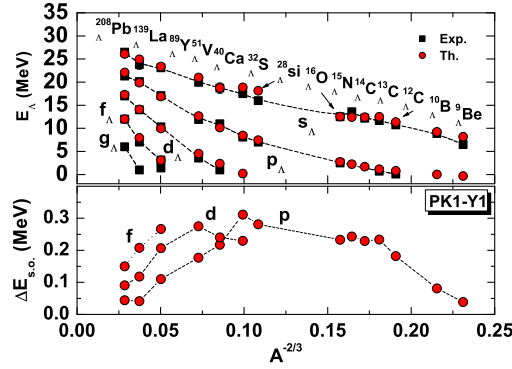


Fig. 1. Single- $\Lambda$  binding energies (upper panel) and spin-orbit splitting sizes (lower panel) for  $\Lambda$  states in RMF calculations with newly-adjusted PK1-Y1 effective interaction. For comparison, the experimental data<sup>5–12</sup> are given as well.

Figure 1 shows the single- $\Lambda$  binding energies and spin-orbit splitting sizes for  $\Lambda$  states with different orbital angular momentum obtained from RMF calculations with PK1-Y1 effective interaction. In the upper panel,

good agreement for single- $\Lambda$  binding energies has been achieved by the theoretical calculations. For the  $\Lambda$  spin-orbit splitting, different from that for nucleon, the magnitude of around several hundreds keV has been found in the calculation. The splitting in medium-mass region are relatively larger than those in light- and heavy- mass regions.

#### 4. Magnetic moments of $\Lambda$ -hypernuclei

With fast development of experimental techniques, the interest in hypernuclear magnetic moments is evoked. The effects of core polarization and tensor coupling on the magnetic moments in  ${}_{\Lambda}^{13}\text{C}$ ,  ${}_{\Lambda}^{17}\text{O}$ , and  ${}_{\Lambda}^{41}\text{Ca}$  hypernuclei are studied in the Dirac equation with scalar, vector and tensor potentials.<sup>49</sup> It is shown that the inclusion of a  $\Lambda$  tensor coupling will modify the current vertex and suppress the effect of core polarization on the magnetic moments. However, as the hyperon wave functions are not sensitive to the  $\Lambda$  tensor potential, the magnetic moments with or without  $\Lambda$  tensor potential are almost the same. The deviations of magnetic moments for  $\Lambda$  in  $p$  states from the Schmidt values are found to increase with the nuclear mass number.

However, this study is based on the perturbation theory for the symmetric nuclear matter. A self-consistent calculation in finite hypernuclei is required, in which both the nucleons and hyperon are treated on the same footing. A self-consistent time-odd triaxial RMF approach<sup>50</sup> include the hyperon and the tensor coupling is developed and applied to study the magnetic moments in hypernuclei.<sup>51</sup> The magnetic moments of  ${}_{\Lambda}^{16-18}\text{O}$  by time-odd triaxial RMF approach with PK1 and PK1-Y1 are shown in Table 2. It is found that the core polarization effect of valence  $\Lambda$  is very important although it is smaller than that of the valence neutron. Furthermore, the core polarized Dirac magnetic moment might be reduced by the tensor coupling of the valence  $\Lambda$ .

#### 5. Nucleus with anti-Lambda

In Ref.,<sup>52</sup> the anti-nucleon spectrum has been studied for ordinary nuclei with the RMF theory and the spin symmetry is found for the single anti-nucleon spectra, i.e., the spin partner states are nearly degenerate and the dominant components of the wave functions are almost the same. It is therefore worthwhile to examine the spin symmetry in single  $\bar{\Lambda}$  spectra.

By taking  ${}^{16}\text{O}$  system as the representative case, the single  $\bar{\Lambda}$  spectra and the  $\bar{\Lambda}$  wave functions were studied in Ref.<sup>53</sup> In the Dirac equation of  $\bar{\Lambda}$ , the

Table 2. The magnetic moments of oxygen hypernuclei in units of nucleon magneton (n.m.), by time-odd triaxial RMF approach with PK1 and PK1-Y1. The total magnetic moment  $\mu_{\text{tot.}}$  is given by the sum of  $\mu_D$ , the anomalous magnetic moment of the nuclear core  $\mu_a^{\text{n+p}}$  and hyperon magnetic moment  $\mu_a^\Lambda$ . While the Schmidt magnetic moment is represented by  $\mu_{\text{Sch.}}$ .

Sys.	$\mu_D$	$\mu_a^{\text{n+p}}$	$\mu_a^\Lambda$	$\mu_{\text{tot.}}$	$\mu_{\text{Sch.}}$
$^{15}\text{O} + \text{free } \Lambda$	-0.113	0.677	-0.613	-0.049	0.025
$^{16}_{\Lambda}\text{O}(\text{Y1})$	-0.132	0.681	-0.610	-0.060	0.025
$^{16}\text{O} + \text{free } \Lambda$	0.	0.	-0.613	-0.613	-0.613
$^{17}_{\Lambda}\text{O}(\text{Y1})$	-0.005	0.	-0.610	-0.614	-0.613
$^{17}\text{O} + \text{free } \Lambda$	-0.134	-1.863	-0.613	-2.610	-2.526
$^{18}_{\Lambda}\text{O}(\text{Y1})$	-0.146	-1.862	-0.610	-2.618	-2.526

scalar and vector potentials of  $\bar{\Lambda}$  are written respectively as  $S_{\bar{\Lambda}}(r) = g_{\sigma\bar{\Lambda}}\sigma$  and  $V_{\bar{\Lambda}}(r) = g_{\omega\bar{\Lambda}}\omega_0$ . The charge conjugation leaves the scalar potential invariant,  $S_{\bar{\Lambda}}(r) = S_{\Lambda}(r)$ , and changes the sign of the vector potential,  $V_{\bar{\Lambda}}(r) = -V_{\Lambda}(r)$ .

In Figure 2 are shown the spin-orbit splittings  $\epsilon_A(nl_{l-1/2}) - \epsilon_A(nl_{l+1/2})$  of anti-Lambda and anti-neutron as functions of the average energy for spin partners in  $^{16}\text{O}$ . The values of splitting for anti-Lambda (0.1 ~ 0.8 MeV) are smaller than those of anti-neutron (0.2 ~ 1.9 MeV), which implies that the spin symmetry in anti-Lambda spectra is even better conserved than that in anti-neutron spectra. It is also found that the dominant components of  $\bar{\Lambda}$  Dirac spinors are almost identical for spin partner states.

The self-consistent effects caused by the  $\bar{\Lambda}$  had not been taken into account in the above calculations. For a real  $\bar{\Lambda}$ - $^{16}\text{O}$  system, the self-consistent mean fields including the scalar and vector ones will be modified by the  $\bar{\Lambda}$ . Further investigation on this issue is in progress.

## 6. Summary

Several aspects about hypernuclei investigated with the relativistic mean field theory, including the effective  $\Lambda$ -nucleon coupling strengths based on the effective nucleon-nucleon interaction PK1, hypernuclear magnetic moment and spin symmetry in  $\bar{\Lambda}$ -hypernuclei have been presented. With the newly-adjusted PK1-Y1 effective interaction, the single- $\Lambda$  binding energies of hypernuclei from light to heavy mass regions have been well reproduced. The effects of tensor coupling and core polarization from valence  $\Lambda$  in  $\Lambda$ -

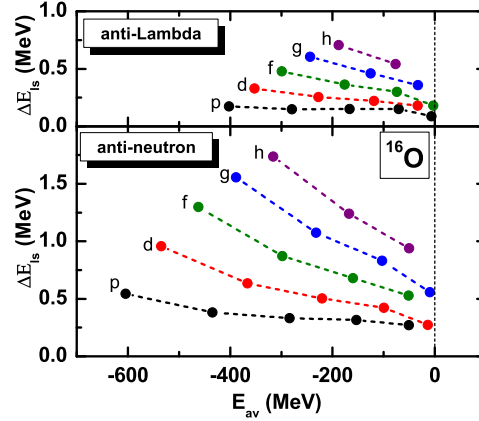


Fig. 2. Spin-orbit splitting  $\epsilon_A(nl_{l-1/2}) - \epsilon_A(nl_{l+1/2})$  in the spectra of anti-Lambda and anti-neutron in  $^{16}\text{O}$  versus the average energy of a pair of spin doublets. The vertical dashed line shows the continuum limit.

hypernuclei have been found to be of importance in the description of hypernuclear magnetic moments. The spin symmetry in  $\bar{\Lambda}$  spectra have been found to be even better developed than that in anti-neutron spectra. The investigation for spin symmetry in single- $\bar{\Lambda}$  spectra in  $\bar{\Lambda}$ -hypernuclei is in progress.

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### References

1. M. Danysz and J. Pniewski *Phil. Mag.* **44** (1953) 348.
2. R. H. Dalitz *et al*, *Proc. R. Soc. London A* **426** (1989) 1.
3. P. H. Pile *et al*, *Phys. Rev. Lett.* **66** (1991) 2585.
4. H. Hotchi *et al*, *Phys. Rev. C* **64** (2001) 044302.
5. D. H. Davis and J. Pniewski, *Contemp. Phys.* **27**, 91 (1986).
6. H. Hotchi *et al*, *Phys. Rev. C* **64**, 044302 (2001).
7. P. H. Pile *et al*, *Phys. Rev. Lett.* **66**, 2585 (1991).
8. T. Nagae, *Nucl. Phys.* **A670**, 269c (2000).
9. T. Hasegawa *et al*, *Phys. Rev. Lett.* **74**, 224 (1995); *Phys. Rev. C* **53**, 1210 (1996).
10. H. Tamura, *Eur. Phys. J.* **A13**, 181 (2002).

11. S. Ajimura *et al.*, *Phys. Rev. Lett.* **86**, 4255 (2001).
12. M. Ukai *et al.*, *Phys. Rev. Lett.* **93**, 232501 (2004).
13. M. Danysz *et al.*, *Nucl. Phys.* **49** (1963) 121; R. H. Dalitz *et al.*, *Proc. R. Soc. London A* **426** (1989) 1.
14. D. Prowse *et al.*, *Phys. Rev. Lett.* **17** (1966) 782.
15. S. Aoki *et al.*, *Prog. Theor. Phys.* **85** (1991) 1287.
16. J. K. Ahn *et al.*, *Phys. Rev. Lett.* **87** (2001) 132504-1.
17. H. Takahashi *et al.*, *Phys. Rev. Lett.* **87** (2001) 212502-1.
18. M. Prakash *et al.*, *Nucl. Phys. A* **639** (1998) 433c.
19. B. X. Sun *et al.*, *Commun. Theor. Phys.* **36** (2001) 446.
20. L. Majling *et al.*, *Nucl. Phys. A* **585** (1995) 211c.
21. I. N. Filikhin *et al.*, *Phys. Rev. C* **65** (2002) 041001.
22. R. H. Dalitz and A. Gal, *Ann. Phys.* **116** (1978) 167.
23. D. E. Lanskoy *et al.*, *Phys. Rev. C* **58** (1998) 3351.
24. B. D. Serot *et al.*, *Adv. Nucl. Phys.* **16** (1986) 1.
25. P-G Reinhard, *Rep. Prog. Phys.* **52** (1989) 439.
26. P. Ring, *Prog. Part. Nucl. Phys.* **37** (1996) 193.
27. J. Meng *et al.*, *Prog. Part. Nucl. Phys.* **57** (2006) 470.
28. J. Boguta *et al.*, *Phys. Lett. B* **102** (1981) 93.
29. R. Brockmann *et al.*, *Nucl. Phys. A* **355** (1981) 365.
30. M. Rufa *et al.*, *J. Phys. G* **13** (1987) 143.
31. J. Mareš *et al.*, *Z. Phys. A* **333** (1989) 209; *Phys. Lett. B* **249** (1990) 181.
32. M. Rufa *et al.*, *Phys. Rev. C* **42** (1990) 2469.
33. M. Chiapparini *et al.*, *Nucl. Phys. A* **529** (1991) 589.
34. J. Schaffner *et al.*, *Phys. Rev. C* **46** (1992) 322.
35. J. Mareš *et al.*, *Phys. Rev. C* **49** (1994) 2472.
36. J. Schaffner *et al.*, *Ann. Phys. (N.Y.)* **235** (1994) 35.
37. Z. Y. Ma *et al.*, *Nucl. Phys. A* **608** (1996) 305.
38. D. Vretenar *et al.*, *Phys. Rev. C* **57** (1998) R1060.
39. J. Meng *et al.*, *Phys. Rev. Lett.* **81** (1998) 460.
40. J. Meng *et al.*, *Phys. Rev. C* **65** (2002) R041302.
41. H. F. Lü *et al.*, *Chin. Phys. Lett.* **19** (2002) 1775.
42. H. F. Lü *et al.*, *J. Eur. Phys. A* **17** (2003) 19.
43. G. A. Lalazissis *et al.*, *Phys. Rev. C* **55** (1997) 540.
44. W. H. Long *et al.*, *Phys. Rev. C* **69** (2004) 034319.
45. Y. Sugahara *et al.*, *Nucl. Phys. A* **579** (1994) 557.
46. J. Cohen and H. J. Weber, *Phys. Rev. C* **44**, 1181 (1991).
47. H. F. Lü *et al.*, to be published.
48. M. M. Sharma *et al.*, *Phys. Lett. B* **312** (1993) 377.
49. J. M. Yao *et al.*, *Chin. Phys. Lett.* **25** (2008) 1629.
50. J. M. Yao *et al.*, *Phys. Rev. C* **74** (2006) 024307.
51. H. F. Lü *et al.*, to be published.
52. S. G. Zhou *et al.*, *Phys. Rev. Lett.* **91** (2003) 262501.
53. C. Y. Song, J. M. Yao, and J. Meng, *Chin. Phys. Lett.* **26** (2009) 122102.